Critical finite-size scaling of energy and lifetime probability distributions of auroral emissions

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[1] Based on statistical study of approximately 15,500 ultraviolet images of auroral emission regions provided by the UVI experiment on the POLAR spacecraft, we show that energy and duration probability distributions of particle precipitation events obey finite-size scaling relations indicative of a self-organized critical (SOC) dynamical state. The revealed relations are invariant with respect to significant changes in the spatial scale of the emission areas, and involve a set of mutually consistent critical exponents providing a quantitative basis for future theoretical studies of multiscale magnetospheric fluctuations. The reported statistical results highlight the importance of cross-scale coupling in the development of nighttime geomagnetic disturbances and suggest that various manifestations of substorm activity associated with localized magnetic reconnections in the magnetotail (small to large scale substorms, pseudo-breakups, BBFs and other types of short-term localized excitations) can be coordinated on the global scale by universal dynamical principle represented by scale-free avalanching in numerical SOC models.
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1. Introduction

[2] It has been shown [*Uritsky et al.*, 2003, 2002] that the spatiotemporal evolution of auroral emission regions observed by the POLAR UVI instrument exhibits several power-law statistical relations. Considered in the context of ongoing studies of multiscale magnetospheric complexity [e.g., *Angelopoulos et al.*, 1999; *Chang et al.*, 2004; *Chapman et al.*, 1999; *Consolini and Chang*, 2001; *Hnat et al.*, 2003; *Kozelov et al.*, 2004; *Lui et al.*, 2000; *Uritsky and Pudovkin*, 1998; *Watkins et al.*, 1999], these relations strongly suggest that Earth's magnetosphere, as an open dissipative dynamical system, operates in a stationary critical thermodynamic state – the state of self-organized criticality (SOC) [*Bak et al.*, 1988; *Chang*, 1992]. Numerical modeling studies [*Klimas et al.*, 2004, 2005] have shown that this state can be supported through cooperative interactions of localized magnetic reconnection regions in the magnetotail. An *in situ* verification of the presence of SOC in the plasma sheet would require extensive simultaneous multi-point measurements that would present a challenging task for future multi-spacecraft missions such as the Magnetospheric Constellation. Until then, satellite-based imaging of auroral particle precipitation will remain the most detailed source of spatiotemporal information on multiscale processes in the tail that may be associated with its SOC state.

[3] In our previous works [*Uritsky et al.*, 2003, 2002], we have adapted numerical methods used for quantifying SOC behavior in discrete numerical models to an extensive statistical investigation of the auroral emission dynamics. Specifically, we have examined the spatiotemporal evolution of auroral emissions over a broad, fixed portion of the night-side auroral region deemed most sensitive to magnetotail activity. Consequently, we have included quite different types of high-latitude activity, such as due to substorms of various sizes, to pseudo-breakups, and to small localized excitations. This approach allowed us to obtain a set of probability distributions that by their construction were analogous to the avalanche distributions in SOC models. However, from the point of view on the magnetospheric physics it would be of crucial importance to trace the contributions of physically different types of perturbations to the total statistics. If the SOC hypothesis is correct, then all subgroups of the observed events should exhibit scale-free spatiotemporal behavior described by the same critical exponents, and so removing different types of events from the overall statistics should not change its power-law characteristics. Here, we confirm this conjecture by presenting the results of a direct statistical test for the invariance of particle precipitation event probability distributions with respect to dramatic changes of the maximum spatial scales of the events.

[4] Our analysis is based on the finite-size scaling (FSS) approach adopted from the theory of critical phenomena and commonly used for verification of the SOC state in numerical models of multiscale turbulence. We impose a maximum size on the activity in a fixed night-side region of the aurora that we accept in our analysis, and study the effects of varying that maximum size. As we show below, the energy and lifetime statistics of auroral emissions can be rather accurately described by the FSS relations within quite a wide range of cutoff scales. The results indicate that probability distributions of selected events have power-law form, with large-scale cutoffs diverging with the maximum size in a way typical for SOC systems.

2. Method and Results

[5] Our analysis of FSS effects in auroral emission dynamics is based on an automated technique for spatio-

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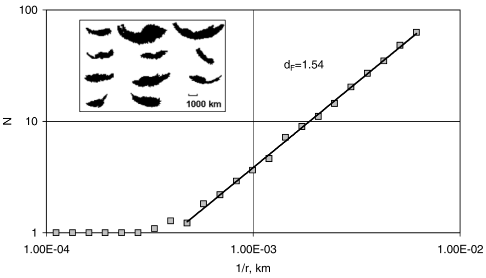

Figure 1. Box-counting statistics of 20 representative emission events (number N of non-overlapping square boxes required to cover the emission region as a function of inverse linear box size r). The power-law shape of the obtained dependence and the fractional value of the dimension d_F suggest that the emission regions have a scale-invariant fractal shape in the range $r = 1.5 \cdot 10^2$–$2.5 \cdot 10^3$ km. Inset: examples of auroral emission regions included in the statistics.

temporal detection of auroral precipitation events. The technique treats the emission events, in analogy with SOC avalanches on a 2-D numerical grid, as three-dimensional objects described by two spatial dimensions given by geomagnetic coordinates and one temporal dimension given by the time axis [*Uritsky et al.*, 2002]. This method was applied to a database of about 15,500 digital UVI images obtained by the UVI experiment on the POLAR spacecraft in the LBH-long wavelength band [*Brittnacher et al.*, 1997] during the period 01 January – 28 February, 1997 with a typical 184-second temporal resolution. Our analysis was focused on the nighttime sector of the aurora (55 to 90 degrees Mlat, 2000 to 0400 MLT) associated with the magnetotail dynamics. The images were transformed into the corrected magnetic coordinate system and coarse-grained with the uniform resolution of 70×70 km^2 to eliminate the difference in pixel sizes at different spacecraft altitudes and to minimize the effects of the wobbling of the satellite spin axis.

[6] Initial spatial locations of the emission regions were detected by applying a lower UVI luminosity threshold of 10 photons$\cdot$cm$^{-2}\cdot$s^{-1}. Then, by checking the intersections (in terms of common pixels) of active regions above this threshold in every pair of consecutive UVI frames, spatio-temporal traces of each of the detected emission events were identified. Only events that lasted longer than the sampling time of 184 s and were not truncated by the edges of the field of view or by time gaps in POLAR UVI observations were included. In total, about 6750 auroral emission events were recognized using this technique. A detailed description of physical and statistical criteria used for selecting emission events for our database, as well as the discussion of their relevance to large-scale magnetospheric activity and substorm phases can be found in [*Uritsky et al.*, 2006, 2002]. The events were characterized by the lifetime T, defined as the time difference between the last and the first UVI image in which an event is present, as well as the total energy deposition E, obtained by integrating the UVI luminosity over the spatiotemporal domain of an event and expressing the result in units of energy of precipitating electrons [*Brittnacher et al.*, 1997].

[7] To experimentally check the validity of the FSS relations, one needs to vary the upper linear scale of the dissipation events. In numerical studies, this task is usually carried out by changing the size of the simulation grid and comparing the shape of the probability distribution functions of the events obtained on those grids. In reality, the overall system size is usually fixed, and one should filter the entire database of observed events by applying an artificial upper linear scale L so that only events with the linear scale $l < L$ are included in statistical analysis. If the emission dynamics is scale-free, it is expected that the probability distributions of filtered data would follow the FSS scaling form [*Bak et al.*, 1988; *Robinson*, 1994]:

$$p(x, l < L) = x^{-\tau_x} f_x(x/x_c), \qquad x_c \sim L^{D_x}, \qquad x \in \{E,\ T\} \tag{1}$$

Here τ_x and D_x are the avalanche and the FSS critical exponents respectively; x_c represents the cutoff scales of the energy and lifetime probability distributions of the emission events. The cutoff effects are described by the functions f_x that decay rapidly if $x > x_c$ and remain constant otherwise.

[8] Due to the irregular and anisotropic shapes of active emission regions (see inset in Figure 1), their linear scale l is not always well defined. A more reliable statistical measure of spatial extent of an emission event is its area s. Assuming that outer boundaries of the auroral active regions have a fractal geometry, the relationship between l and s can be written as

$$l = a\, s^{1/d_F} \tag{2}$$

where d_F is the fractal dimension and a is some constant. To verify this scaling law and estimate d_F, we have studied 20 representative emission events using the method of box statistics [*Turcotte*, 1997]. The method consists of counting the number N of non-overlapping square boxes of size r required to cover the fractal image under consideration. By varying r and plotting N versus $1/r$, one evaluates the fractal dimension, which is equal to the power-law exponent of this dependence. We have found (Figure 1) that the box-counting statistics of emission events has a distinct power-law form (2) ranging over about an order of magnitude in r and described by the fractal dimension $d_F = 1.54 \pm 0.02$. The coefficient a was calculated under the assumption that the relation remains valid for the smallest available spatial scale $l_0 = 70$ km, which gives $a = l_0^{1-2/d_F} \approx 0.281$.

[9] By varying the maximum emission area within the range $1.0 \cdot 10^4$ to $6.4 \cdot 10^5$ km^2, we varied the upper linear scale L between $1.1 \cdot 10^2$ and $1.7 \cdot 10^3$ km. Figure 2 shows the probability distributions of emission events over their energy and lifetime obtained for different L values. As L decreases, the cutoff scale of both distributions shifts to the left, and their power-law shapes get more and more distorted. It turns out, however, that this distortion follows very closely the scaling form (1) indicative of scale-free fluctuations in critical systems.

[10] The FSS ansatz (1) assumes that the cutoff functions f_x have the following distinctive features: (a) their analytical forms are *invariant* under the transformation of the upper linear scale of the auroral precipitation regions; (b) they remain nearly constant in the range $l_0 < l < L$ where auroral dynamics is not affected by the scale limitation; (c) they

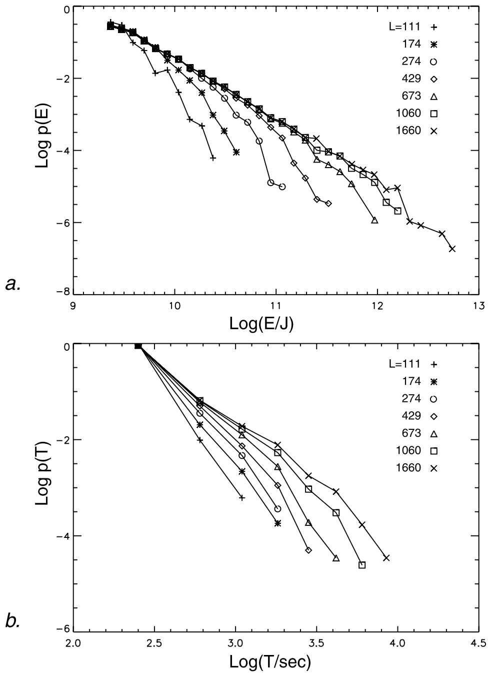

Figure 2. Probability distributions of auroral emission events over (a) energy and (b) lifetime for different maximum linear sizes L (notation Log is used for base-10 logarithms). The original distributions constructed without applying the size limitation are described by power law exponents $\tau_E \approx 1.60$ and $\tau_T \approx 2.24$.

exhibit a fast decay as l becomes comparable with L. To check these features, we have rescaled the probability distributions shown in Figure 2 by plotting them in the transformed coordinate system

$$X = \frac{x}{L^{D_x}} \sim x/x_c, \;\; Y = p(x)\; x^{\tau_x} \equiv f_x(x/x_c). \tag{3}$$

In this transformation, we used the values of the auroral emission exponents τ_T and τ_E reported in [*Uritsky et al.*, 2002]. The exponent D_E was calculated from the least-squares approximation of the $E(l)$ scatter plot by the power-law relation

$$E \sim l^{D_E} \tag{4}$$

predicted for SOC dynamics. We found this relation to be applicable within almost the entire range of spatial scales of interest (Figure 3) and obtained $D_E = 2.14 \pm 0.03$. Due to significantly weaker correlation between T and L (presumably because of insufficient sampling frequency of UVI images), the exponent D_T could not be derived from a similar regression dependence. Instead, we considered this exponent as a tuning parameter in the FSS renormalization (3) which has revealed that the best "collapse" of the rescaled lifetime distributions occurs at D_T = 1.02. The obtained estimate agrees with the value 1.1 that we obtained using an independent method based on the theoretical relation for the dynamic critical exponent $D_T = \alpha/\beta$ in which α and β are the power-law exponents describing respectively spatial and temporal scaling of the second-order structure function of the auroral luminosity (to be published elsewhere).

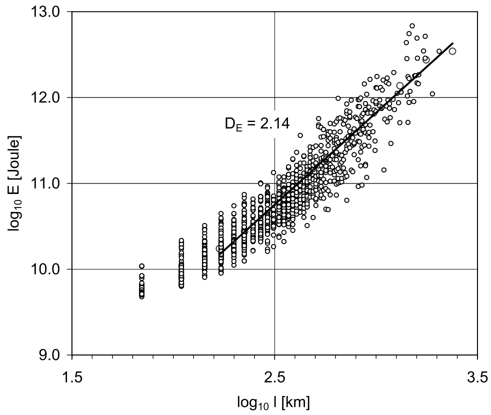

Figure 3. Scatterplot of emission energies versus emission linear size used for evaluating the FSS power-law exponent D_E. The exponent has been calculated within the range of linear scales l corresponding to the range of fractal behavior of emission region boundaries revealed by the box-counting statistics (see Figure 1).

[11] As can be seen from Figure 4, the rescaled versions of $p(T)$ and $p(E)$ distributions fall on approximately the same functional curves independent of the cutoff length L. It can also be noticed that these curves have the expected shape with nearly constant and rapidly decreasing regions at small and large values of the rescaled arguments, correspondingly. These observations indicate that the auroral

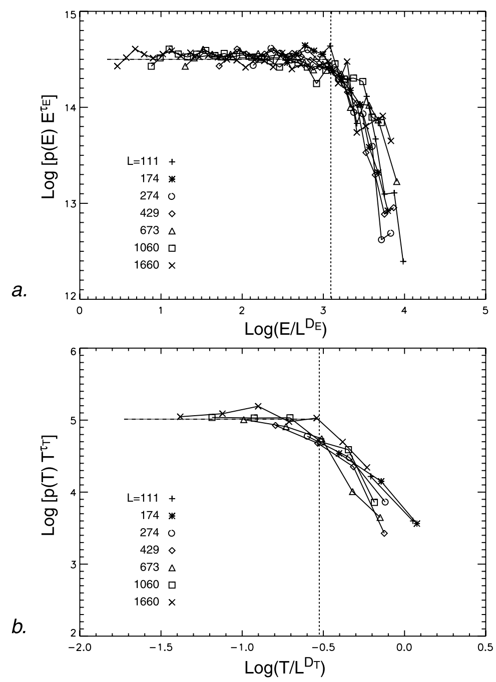

Figure 4. Rescaled versions of the probability distributions from Figure 2 showing the invariant shape of the FSS cutoff functions f_E and f_T under the transformation of the cutoff scale L. The vertical and the horizontal dotted lines show the cutoff scales and the average values of each distribution below their cutoff scales, respectively. The observed data collapse confirms the correctness of our estimation of the scaling exponents and reveals a scale-free nature of auroral emission dynamics.

emission dynamics does obey the FSS and therefore has no intrinsic spatial scales. The only scale that is present in our analysis, and that has been successfully eliminated after the renormalization (3), is the artificial maximum length L. The observed data collapse also confirms the correctness of the involved exponents, all of which except for D_T have been evaluated independently of the FSS analysis. Indeed, we have found that the rescaled distributions shown in Figure 4 are very sensitive to the τ_E, τ_T, D_E and d_F values and do not match if these exponents are shifted from their experimental values by more then 5%.

3. Discussion and Conclusions

[12] We have found that the finite-size renormalization (3), which depends on a combination of several critical exponents, provides a remarkable "data collapse" of the probability distribution functions constructed for a broad range of maximum length scales of auroral emission regions. This result is fully consistent with the SOC hypothesis as it strongly suggests that the magnetosphere is a scale-free critical system whose unloading dynamics contains no distinct characteristic energy or time scales except for the scales imposed by its global relaxation time and the overall energy resource. Physically, the applicability of the FSS form to auroral brightenings indicates strong and robust coupling between magnetospheric disturbances of quite different scales. This coupling may be due to a multiscale self-organization of sporadic localized reconnections that, according to recent simulation results [*Klimas et al.*, 2004, 2005], can drive the magnetotail plasma sheet toward a globally stable SOC state whose critical features are similar to those of multiscale electron precipitation regions in the nighttime aurora.

[13] From a more general perspective, the broad-band scale-invariant nature of the statistical laws reported in this letter appears to jeopardize the common reductionist view of substorm activity as a combination of several distinct classes of magnetospheric disturbances. Our results suggest a more generic picture in which various magnetospheric effects such as small to large scale substorms, pseudo-breakups, short-term localized excitations associated with bursty bulk flows and other small-sale disturbances, actually develop and organize according to the same global thermodynamic principle described by the theory of SOC and manifested in the universal tendency of driven many-body dynamical system toward a globally stable critical point [*Bak*, 1997; *Jensen*, 1998]. On a qualitative level, this principle implies that Earth's magnetosphere is functioning as a holistic system rather than a collection of separate physical mechanisms controlling plasma dynamics at individual scales. Although these mechanisms take important parts in generating specific types of magnetospheric disturbances, it is their inherent cross-scale organization that controls the spatial and temporal variations of the resulting energy output into the high-latitude ionosphere. We hope that our observations, along with other studies of scale invariance in magnetospheric activity, will stimulate interest in this largely unexplored area.

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